INTRODUCTION TO THREE-NEUTRINO MIXING PARAMETERS LISTINGS

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Introduction and Notation:

With the exception of the LSND anomaly, current accelerator, reactor, solar and atmospheric neutrino data can be described within the framework of a 3×3 mixing matrix between the flavor eigenstates ν_e, ν_μ and ν_τ and mass eigenstates ν_1, ν_2 and ν_3. (See equation 13.77 of the review "Neutrino Mass, Mixing and Oscillations" by K. Nakamura and S.T. Petcov.) Whether or not this is the ultimately correct framework, it is currently widely used to parametrize neutrino mixing data and to plan new experiments.

The mass differences are called $\Delta m^2_{21} \equiv m^2_2 - m^2_1$ and $\Delta m^2_{32} \equiv m^2_3 - m^2_2$. In these listings, we assume

$$\Delta m^2_{32} \sim \Delta m^2_{31} \tag{1}$$

although in the future, experiments may be precise enough to measure these separately. The angle are labeled θ_{12}, θ_{23} and θ_{13}. The CP violating phase is called δ, but that does not yet appear in the listings. The familiar two neutrino form for oscillations is

$$P(\nu_a \to \nu_b) = \sin^2(2\theta)\sin^2(\Delta m^2 L/4E). \tag{2}$$

Despite the fact that the mixing angles have been measured to be much larger than in the quark sector, the two neutrino form is often a very good approximation and is used in many situations.

The angles appear in the equations below in many forms. They most often appear as $\sin^2(2\theta)$. The listings currently use this convention.

Accelerator neutrino experiments:

Ignoring the small Δm^2_{21} scale, *CP* violation, and matter effects, the equations for the probability of appearance in an accelerator oscillation experiment are:

$$P(\nu_\mu \to \nu_\tau) = \sin^2(2\theta_{23})\cos^4(\theta_{13})\sin^2(\Delta m^2_{32}L/4E) \quad (3)$$
$$P(\nu_\mu \to \nu_e) = \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2(\Delta m^2_{32}L/4E) \quad (4)$$
$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2(\Delta m^2_{32}L/4E) \quad (5)$$
$$P(\nu_e \to \nu_\tau) = \sin^2(2\theta_{13})\cos^2(\theta_{23})\sin^2(\Delta m^2_{32}L/4E)\ . \quad (6)$$

For the case of negligible θ_{13}, these probabilities vanish except for P$(\nu_\mu \to \nu_\tau)$, which then takes the familiar two-neutrino form.

New long-baseline experiments are being planned to search for non-zero θ_{13} through $P(\nu_\mu \to \nu_e)$. Including the CP violating terms and low mass scale, the equation for neutrino oscillation in vacuum is:

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= P1 + P2 + P3 + P4 \\ P1 &= \sin^2(\theta_{23})\sin^2(2\theta_{13})\sin^2(\Delta m^2_{32}L/4E) \\ P2 &= \cos^2(\theta_{23})\sin^2(2\theta_{13})\sin^2(\Delta m^2_{21}L/4E) \\ P3 &= -/+\, J\sin(\delta)\sin(\Delta m^2_{32}L/4E) \\ P4 &= J\cos(\delta)\cos(\Delta m^2_{32}L/4E) \end{aligned} \quad (7)$$

where

$$\begin{aligned} J = &\cos(\theta_{13})\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\times \\ &\sin(\Delta m^2_{32}L/4E)\sin(\Delta m^2_{21}L/4E) \end{aligned} \quad (8)$$

and the sign in P3 is negative for neutrinos and positive for anti-neutrinos. For most new proposed long-baseline accelerator experiments, P2 can safely be neglected, but depending on the values of θ_{13} and δ, the other three terms could be comparable. Also, depending on the distance and the mass hierarchy, matter effects will need to be included.

Reactor neutrino experiments:

Nuclear reactors are prolific sources of $\bar{\nu}_e$ with an energy near 4 MeV. The oscillation probability can be expressed

$$\begin{aligned} P(\bar{\nu}_e \to \bar{\nu}_e) = 1 &- \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2(\Delta m^2_{21}L/4E) \\ &- \cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta m^2_{31}L/4E) \\ &- \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta m^2_{32}L/4E) \end{aligned} \quad (9)$$

not using the approximation in Eq. (1). For short distances (L<5 km) we can ignore the second term on the right and can reimpose approximation Eq. (1). This takes the familiar two neutrino form with θ_{13} and Δm^2_{32}:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2(2\theta_{13}) \sin^2(\Delta m^2_{32} L/4E). \qquad (10)$$

For long distances and small θ_{13}, the last two terms in Eq. (9) oscillate rapidly and average to zero for an experiment with finite energy resolution, leading to the familiar two neutrino form but with θ_{12} and Δm^2_{21}.

Solar and Atmospheric neutrino experiments:

Solar neutrino experiments are sensitive to ν_e disappearance and have allowed the measurement of θ_{12} and Δm^2_{21}. They are also sensitive to θ_{13}. We identify $\Delta m^2_{\odot} = \Delta m^2_{21}$ and $\theta_{\odot} = \theta_{12}$.

Atmospheric neutrino experiments are primarily sensitive to ν_μ disappearance through $\nu_\mu \rightarrow \nu_\tau$ oscillations, and have allowed the measurement of θ_{23} and Δm^2_{32}. We identify $\Delta m^2_A = \Delta m^2_{32}$ and $\theta_A = \theta_{23}$. Despite the large ν_e component of the atmospheric neutrino flux, it is difficult to measure Δm^2_{21} effects. This is because of a cancellation between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ together with the fact that the ratio of ν_μ and ν_e atmospheric fluxes, which arise from sequential π and μ decay, is near 2.

Oscillation Parameter Listings:

In Section (B) we encode the three mixing angles θ_{12}, θ_{23}, θ_{13} and two mass squared differences Δm^2_{21} and Δm^2_{32}. Our knowledge of θ_{12} and Δm^2_{21} comes from the KamLAND reactor neutrino experiment together with solar neutrino experiments. Our knowledge of θ_{23} and Δm^2_{32} comes from atmospheric neutrino experiments and long-baseline accelerator experiments. Searches for a non-zero value of θ_{13} are proceeding at reactor experiments looking for $\bar{\nu}_e$ disappearance and at long-baseline accelerator experiments looking for ν_e appearance. The interpretation of both kinds of results depends on Δm^2_{32}, and the accelerator results also depend on the mass hierarchy, θ_{23} and the CP violating phase δ. We present 90%CL limit on θ_{13} at the current best fit value of Δm^2_{32}, but that limit is asymmetric around that best fit value. There is a 50% chance that the upper limit is higher. A true 90%CL upper limit cannot be calculated without a global fit (we note that the union of two Confidence Levels is not a Confidence Level). A more conservative approach would be to quote the reactor 90% CL at the one sigma low value for Δm^2_{32} and that is done in the footnote when possible.