EXTRA DIMENSIONS

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I. Introduction

The large separation between the weak scale $\sim 10^3$ GeV and the traditional scale of gravity—the Planck scale with $M_{\rm Pl} \sim 10^{19}$ GeV—is one of the most puzzling aspects of nature. The origin of this large ratio, as well as its stability under radiative corrections, demands explanation. This is known as the hierarchy problem. One theoretical means of solving this problem is to introduce Supersymmetry (see the "Note on Supersymmetry" in this Review). Alternatively one may hope to address the hierarchy by exploiting the geometry of space time. Specifically, recent theories involve the idea that the 3-spatial dimensions in which we live could be a 3-spatial-dimensional 'membrane' embedded in a much larger extra dimensional space, and that the hierarchy is generated by the geometry of the additional dimensions. Such ideas have led to extra dimensional theories which have verifiable consequences at the TeV scale.

Our knowledge of the weak and strong forces extends down to scales of $\sim (100 \text{ GeV})^{-1}$ (or of order 10^{-15} mm). On the other hand, we have almost no knowledge of gravity at distances less than roughly a *millimeter*, as direct tests of the gravitational force at the smallest distances are based on torsion-balance experiments, which are mechanically limited. It is thus conceivable that gravity may behave quite differently from the 3-dimensional Newtonian theory at small distances.

This leads to the possibility that matter and non-gravitational forces are confined to our 3-dimensional subspace, whereas gravity may propagate throughout a higher dimensional volume. In this case, the gauge forces are trapped within our 3-dimensional space, unaware of the extra dimensions, and maintain their usual behavior. Gravity, on the other hand, would no longer follow the inverse-square force law at distances smaller than the size of the extra dimensions, as the gravitational equivalent of Gauss' Law mandates that the gravitational field spreads out into the full spatial volume.

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Since Newton's Law must be reproduced at large distances, gravity must behave as if there were only three spatial dimensions for $r \gtrsim 1$ mm. This is achievable either by compactifying all the extra dimensions on circles, where the geometry of these dimensions is thus flat and the topology is that of a torus, or by using strong curvature effects in the extra dimensions. In the first case, Arkani-Hamed, Dimopoulos, and Dvali (ADD) [1,2] used this picture to generate the hierarchy by postulating a large volume for the extra dimensional space, building on earlier ideas in Refs. 3,4. In the latter case, the hierarchy can be established by a large curvature of the extra dimensions as demonstrated by Randall and Sundrum (RS) [5,6]. It is the relation of these models to the hierarchy which yields testable predictions at the TeV scale.

General Features of Models

More technically, our subspace of $(3+1)$ space-time dimensions is known as a '3-brane', where the terminology is derived from a generalization of a 2-dimensional membrane. This brane is embedded in a D-dimensional space-time, $D \equiv (3+\delta+1)$, known as the 'bulk'. It is usually assumed that all δ-dimensions transverse to the brane have a common size R. However, the brane can also have smaller extra dimensions associated with it, of size $r \ll R$, and through which it extends, leading to effects similar to a small finite thickness. The size and geometry of the bulk, as well as the types of particles which are allowed to propagate in the bulk and on the brane, vary between different models.

Upon compactification of the δ-additional dimensions, all fields which propagate in the bulk are Fourier expanded into a complete set of modes—the so-called Kaluza-Klein (KK) tower of states, with mode numbers $\vec{n} = (n_1, n_2, \ldots, n_\delta)$ labeling the KK excitations. Similar to a particle in a box, the momentum of the bulk field is quantized in the δ compactified dimensions, given by $\vec{p}_\delta^{\,2} = \vec{n}\cdot\vec{n}/R^2$, where R is the compactification radius. From the 4d perspective of an observer on the brane, each allowed momentum in the compactified volume appears as a KK excitation of the bulk field with mass $m_{\vec{n}}^2 = \vec{p}_\delta^{\,2}$. This builds a KK tower of states where each KK excitation carries identical spin and gauge quantum numbers. Kaluza-Klein states are a generic feature of models with compactified dimensions. The above assumes that all additional dimensions are of the same size and are flat; in more complicated compactifications, the Fourier expansion must be generalized, and the mass formula no longer takes on the above simple form.

The many proposed scenarios may be divided into two categories, depending on whether they do or do not assume that the geometry of the full $(4+\delta)$-dimensional space time with metric G_{IJ} is of factorized form, where the $4d$ and δ-dimensional geometries are independent. In the factorized case, the metric can be put in the form

$$ds^2 = G_{IJ}dx^I dx^J = \eta_{\mu\nu}dx^\mu dx^\nu + h_{ij}(y)dy^i dy^j \ , \qquad (1)$$

where $I, J = (0, \ldots, 3+\delta)$, $\mu, \nu = (0, ..., 3)$ and $i, j = (1, \ldots, \delta)$. The metric h_{ij} for the extra dimensions is flat only if they are toroidal, as assumed in the ADD scenario. In general, however, the bulk geometry is curved, even in the factorized case, and this can have important consequences. In the non-factorizable case, where there is a function of y multiplying $\eta_{\mu\nu}dx^\mu dx^\nu$, the bulk geometry is automatically curved. This is sometimes referred to as a 'warped' geometry.

A further classification involves the field content assumed to be present in the bulk or confined to the brane. The latter may be accomplished via localization, where the field's wavefunction is narrowly peaked about the brane. The low-energy effective action for the bulk contains, at minimum, the higher-dimensional Einstein-Hilbert term

$$S = \int d^4x d^\delta y \sqrt{-\det G} \ \left\{ \frac{\overline{M}_D^{(2+\delta)}}{2} \mathcal{R} + \ldots \right\} \qquad (2)$$

where $\mathcal{R}$ is the $(4+\delta)$-dimensional Ricci scalar. This expression defines the scale $\overline{M}_D$, which is the fundamental scale of the higher-dimensional theory, and is the analog of the 4-d reduced Planck mass, $\overline{M}_{\rm Pl} = 2.4 \times 10^{18}$ GeV. This theory is non-renormalizable, and thus, gravitational interactions grow with energy as $\sim (E/\overline{M}_D)^{(2+\delta)/2}$. Hence, as the energy in a process grows, the theory becomes strongly coupled, and $\overline{M}_D$ is the scale at which the low-energy description breaks down.

This bulk action is incomplete as it does not contain the dynamics that stabilize the extra dimensions at a given size [2,7,8]. This issue is very model-dependent and, with the exception of the warped scenario [5,9], no standard picture exists.

It is common practice to assume that the Standard Model (SM) fields are confined to the brane. A motivation for this assumption is that confinement of certain degrees of freedom (but not gravity in general) to branes is automatic [10] within string theory. However, if the extra dimensions are small enough, the SM fields are phenomenologically allowed to propagate in the bulk, and this possibility allows for novel model-building techniques to address gauge coupling unification, Supersymmetry breaking, the neutrino mass spectrum, and the fermion mass hierarchy.

A general issue that can strongly affect the phenomenology of these scenarios is that branes can be of two types: 'rigid,' or flexible (see Refs. 7,8). If the brane is flexible, it can fluctuate in the extra dimensions, resulting in Nambu-Goldstone modes. These can have important phenomenological implications which are detailed below [11,12,13]. The other possibility is a rigid brane, which can be thought of as a boundary of the bulk space where gravity satisfies particular boundary conditions. In this case the Poincare invariance of the higher dimensional theory is *explicitly* broken and there are no Nambu-Goldstone modes. Due to the broken translational invariance, momentum in the transverse y^i directions is not conserved, and the production of a single bulk mode is allowed. Almost all studies to date consider this 'rigid brane' case.

II. Three scenarios

There are three principal scenarios with predictions at the TeV scale, each of which has a distinct phenomenology.

Large Extra Dimensions

In these theories [1,2], gravity alone propagates in the bulk, where it is assumed to become strong near the electroweak scale. Gauss' Law relates the (reduced) Planck scale of the effective 4d low-energy theory and the fundamental scale $\overline{M}_D$, through the volume of the compactified dimensions, V_δ, via

$$\overline{M}^2_{\rm Pl} = V_\delta \overline{M}_D^{2+\delta}\,. \tag{3}$$

(Here and below we use the conventions of [14].) $M_{\rm Pl}$ is thus no longer a fundamental scale as it is generated by the large volume of the higher dimensional space. If, following ADD, it is assumed that the extra dimensions are *flat*, and thus of toroidal form, then setting $\overline{M}_D \sim$ TeV to eliminate the hierarchy between $M_{\rm Pl}$ and the weak scale determines the compactification radius R of the extra dimensions. Under the simplifying assumption that all radii are of equal size, we can define $V_\delta = (2\pi R)^\delta$. R then ranges from a sub-millimeter to a few fermi for $\delta = 2$–6. The case of $\delta = 1$ is excluded as the corresponding dimension would directly alter Newton's law on solar-system scales. The large size of the additional dimensions forces the SM fields to be constrained to the brane. The bulk graviton expands into a KK tower of spin-2 states which have masses of $\sqrt{\vec{n}^2/R^2}$, where $\vec{n}$ labels the KK excitation level.

The interactions of the bulk graviton with matter and gauge fields are

$$S_{int} = -\frac{1}{\overline{M}_D^{\delta/2+1}} \int d^4x d^\delta y_i h_{AB}(x_\mu, y_i) T_{AB}(x_\mu, y_i)\,, \tag{4}$$

where h_{AB} is the bulk graviton fluctuation, and T_{AB} is the symmetric conserved stress-energy tensor. Setting $T_{AB} = \eta^\mu_A \eta^\nu_B T_{\mu\nu} \delta(y_i)$ for fields on the brane, expanding h_{AB} into the KK tower, and integrating over $d^\delta y_i$ gives the interactions of the bulk KK gravitons with the SM fields on the brane. The Feynman rules governing these interactions are explicitly derived in Refs. 14,15. Each KK excitation state, G_n, couples universally to the Standard Model fields with a strength of $\overline{M}_{\rm Pl}^{-1}$. In addition, a scalar mode exists, which is the volume modulus or radion field representing the fluctuations of the compactification volume. This field couples to the trace of the stress-energy tensor. It is important to note that this description is an effective 4-dimensional theory, valid for energies below $\overline{M}_D$. The full theory above this scale is unknown.

TeV^{-1}-sized Extra Dimensions with SM Fields

The possibility of TeV^{-1}-sized extra dimensions arises in braneworld models [3]. It does not allow for a reformulation of the hierarchy problem. In this case, the Standard Model field content may propagate in the bulk. This allows for several model-building choices: (i) all, or only some, of the SM gauge fields are present in the bulk; (ii) the Higgs field(s) may be in the bulk or on the brane; (iii) the confinement of the SM fermions to the brane or to specific locales in the extra dimensions. If the Higgs field(s) propagate in the bulk, the vacuum expectation value (VEV) of the Higgs zero-mode, the $\vec{n}=0$ KK state, generates spontaneous symmetry breaking. In this case, the gauge boson KK mass matrix is diagonal with the gauge excitation masses being given by $[M_0^2 + \vec{n}\cdot\vec{n}/R^2]^{1/2}$, where M_0 is the VEV-induced mass of the gauge zero-mode. However, if the Higgs is confined to the brane, its VEV induces off-diagonal elements in the mass matrix, generating mixing amongst the gauge KK states of order $(M_0R)^2$. For the case of 1 extra dimension, the coupling strength of the bulk KK gauge states to the SM fermions is $\sqrt{2}g$, where g is the SM SU(2) coupling. The fermion fields may (a) be constrained to the rigid $(3+1)$-brane, in which case they are not directly affected by the extra dimensions; (b) be localized at specific points in the TeV^{-1} dimension, but not on a rigid brane. Here, the zero and excited-mode KK fermions obtain narrow Gaussian-like wave functions in the extra dimensions, with a width much smaller than R^{-1}. This possibility may suppress the rates for a number of dangerous processes such as proton decay [16]; (c) propagate in the bulk. This scenario is known as universal extra dimensions [17]. In this possibility, all fields propagate in the bulk, and thus, branes need not be present. $(4+\delta)$-dimensional momentum is then conserved at tree-level, and KK parity, $(-1)^n$, is conserved to all orders. The phenomenology is quite different for this brane-less theory.

Bulk KK gauge bosons within this context are also discussed in the section on Indirect Constraints on Kaluza-Klein Gauge Bosons in the Listings for "Searches for Heavy Bosons Other Than Higgs Bosons" in this *Review.*

Warped Extra Dimensions

In the simplest form of this scenario [5,6], known as RS1, gravity propagates in a 5d bulk of finite extent, with two rigid boundaries of dimensionality $(3+1)$. The Standard Model fields are assumed to be constrained to one of these rigid $(3+1)$-branes. This configuration permits the metric

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2\,, \tag{5}$$

where the exponential function, or warp factor, multiplying the usual 4d Minkowski term, demonstrates the non-factorizable geometry, and $y \in [0, \pi R]$ is the coordinate of the extra dimension. Here, k describes the curvature scale, which together with $\overline{M}_D$ $(D = 5)$ is assumed [5,6] to be of order $\overline{M}_{\rm Pl}$, with

$$\overline{M}_{\rm Pl}^2 = \frac{\overline{M}_D^3}{k}(1 - e^{-2kR\pi})\,. \tag{6}$$

Consistency of the low-energy description requires that the curvature be small, so $k/\overline{M}_{\rm Pl} \lesssim 0.1$ is assumed. Eq. (5) leads to the gravitational wavefunction being concentrated on the brane at $y = 0$. Moreover, the exponential dependence of proper length and energy scales with y implies that TeV scales are naturally realized and stabilized [9] on the second brane at $y = \pi R$, provided that $kR \simeq 10$. Therefore, the RS1 model [5] localizes the SM fields to this brane at $y = \pi R$. The scale $\Lambda_\pi = \overline{M}_{\rm Pl} e^{-kR\pi} \sim 1$ TeV describes the scale of all physical processes on this so-called 'TeV-brane' at $y = \pi R$. Note that since $kR \simeq 10$ and $k \sim 10^{18}$ GeV is assumed by Randall and Sundrum, this is *not* a model with a large extra dimension.

The 4d phenomenology is governed by the two parameters, Λ_π and $k/\overline{M}_{\rm Pl}$. The masses of the bulk graviton KK tower states on the TeV-brane are $m_n = x_n k e^{-kR\pi} = x_n \Lambda_\pi k/\overline{M}_{\rm Pl}$

with the x_n being the roots of the first-order Bessel function J_1. The KK states are thus not evenly spaced. For typical values of the parameters, the mass of the first graviton KK excitation is of order a TeV. The interactions of the bulk graviton KK tower with the SM fields are [18]

$$\Delta\mathcal{L} = -\frac{1}{M_{\rm Pl}} T^{\mu\nu}(x) h^{(0)}_{\mu\nu}(x) - \frac{1}{\Lambda_\pi} T^{\mu\nu}(x) \sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}(x)\,. \qquad (7)$$

Experiments can determine or constrain the masses m_n and the coupling Λ_π. The couplings of the excitation states are inverse to TeV strength, which results in a different phenomenology than in the case of large extra dimensions.

Extensions of this basic model allow for the SM fields to propagate in the bulk [19,20] since R is small. In this case, the masses of the bulk fermion, gauge, and graviton KK states are related. A third parameter, associated with the fermion bulk mass, is introduced and governs the 4d phenomenology.

An alternate possibility is RS2 [6]; here the SM fields are assumed to live on the brane at $y = 0$, where the graviton zero mode is concentrated, and the second brane is taken off to infinity $R \to \infty$. In this case, there is *no* mass gap in the bulk KK modes, and their coupling to the SM fields on the $y = 0$ brane is much weaker than $1/M_{\rm Pl}$. The collider constraints are investigated in Ref. 21, and cosmological constraints in Ref. 22. Although this setup no longer provides a reformulation of the hierarchy problem, it allows for a modification of gravity, potentially giving signals in sub-mm gravitational force experiments.

III. Experimental constraints

Tests of the Gravitational Force Law

Deviations from the 4d inverse-square gravitational force law may be observable in the case of large flat (ADD) extra dimensions, or in the RS2 scenario. Gravity would obey Gauss' Law in $3+\delta$ spatial dimensions for distances $r < R$ with

$$V_{3+1+\delta}(r) = \frac{-1}{8\pi(2\pi)^\delta(\overline{M}_D)^{\delta+2}} \frac{m_1 m_2}{r^{\delta+1}}\,, \qquad (8)$$

while observing the usual $(\overline{M}_{\rm Pl}^2 r)^{-1}$ gravitational potential for distances $r > R$. The experimental bounds on such deviations [23] are displayed in Fig. 1, which shows the constraints on the general form for the gravitational potential

$$V(r) = -G_N \frac{m_1 m_2}{r} \left(1 + \alpha e^{-r/\lambda}\right) . \tag{9}$$

$\delta = 2$ large extra dimensions predict $\alpha = 4$ for compactification on a torus, which leads to the bound $R < 218\ \mu m$. For $\delta > 2$, R is too small for deviations, due to extra dimensional gravity to be detected in mechanical experiments.

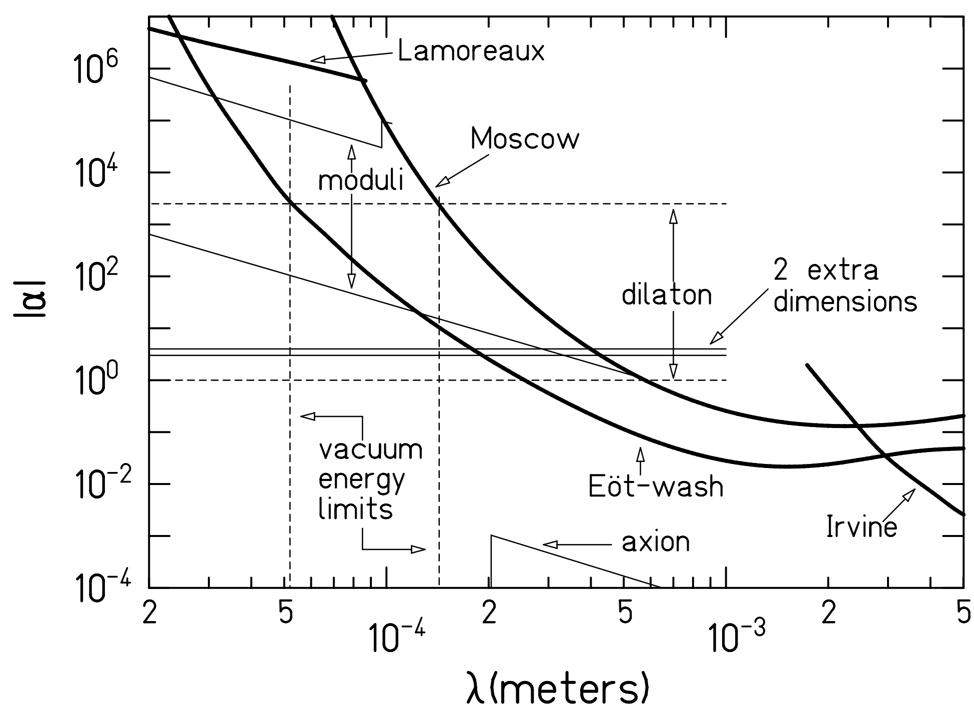

Figure 1: Constraints on deviations from Newton's gravitational force law. The allowed region is below the dark solid lines. From Ref. 23.

Astrophysical and Cosmological Constraints

As first analyzed in Ref. 2, astrophysical and cosmological considerations impose significant constraints on extra-dimensional theories. *Depending on the form of the spectrum of KK excitations of the bulk graviton*, these can imply stringent lower bounds on the gravitational scale $\overline{M}_D$, and corresponding upper bounds on the radii of the extra dimensions, given that Newton's constant is correctly reproduced by Eq. (3). For extra dimensions of the ADD type, these constraints are significantly more stringent than those from direct collider or micro-gravity experiments for $\delta = 2$.

The spectrum of masses of the graviton KK modes sensitively depend on the topology and geometry of the extra dimensions. The spectral quantity of most direct importance for astrophysical and cosmological constraints is the mass gap between the bulk graviton zero mode and the first excited state. For flat (toroidal) extra dimensions of the type considered by ADD, the gap between the zero-mode and first excited state is given by $1/R$, where R is the (assumed common) radius of the extra dimensions, and the number density of KK modes grows as a power law $\rho(k) \sim V_\delta k^{\delta-1}$. Thus, in this case, the KK modes can be extremely light, *e.g.*, $m_n \geq nR^{-1} \geq n \times 10^{-4}$ eV, for $\delta = 2$, and numerous, $N_{KK} \simeq M_{\rm Pl}^2/\overline{M}_D^2 \leq 10^{32}$. As a result, even though these modes are individually only weakly coupled, with strength $1/M_{\rm Pl}$, they can be copiously produced by energetic processes on our brane.

However, such features are model-dependent. For curved extra dimensions, the spectral density of KK modes can possess a large gap, even approaching $\mathcal{O}(30\ \ \text{GeV})$ in extreme cases [24]. Since the typical energy scale in astrophysical and cosmological conditions of interest is at most 100 MeV, the highly curved case avoids the constraints listed below with the exception of A2.

In the warped (RS1) case, these constraints are also avoided, as the scales k and $1/R$ are chosen to be close to the traditional Planck scale, and the resulting spectrum of KK modes has spacing of order a few hundred GeV or greater. In the warped

RS2 case, the cosmological constraints are very mild even for $k \gtrsim 1\ \text{mm})^{-1}$ [22].

We now list the astrophysical and cosmological constraints for *flat extra dimensions* of toroidal type, and *involving solely the bulk graviton* and converted to the conventions of [14]. The constraints can depend upon, (A) only the production of bulk graviton KK modes, such as the case, in the anomalous supernova cooling and overclosure constraints, or (B) both the production and decay back to our brane of the bulk graviton KK modes, such as in the diffuse gamma ray background constraint. The model dependence of the constraints in class (A) is somewhat weaker than for class (B). All bounds become weak for $\delta \geq 4$.

A1 Anomalous cooling of red giants and supernovae due to bulk graviton emission. This was estimated by ADD [2] and calculated in Ref. 25 and further refined in Ref. 26. In particular, the observations of SN1987A place strong constraints on this energy loss mechanism, the dominant astrophysical uncertainty being the unknown core temperature T_{SN1987A} of SN1987A (estimates vary from 30 to 70 MeV). For the case of two flat extra dimensions, a bound of $\overline{M}_D \gtrsim 10\ \text{TeV}(T_{\text{SN1987A}}/30\ \text{MeV})^{1.375}$ is obtained, corresponding to a radius $R \lesssim 7.1 \times 10^{-4}$mm for $T_{\text{SN1987A}} = 30$ MeV. For three extra dimensions $R \lesssim 8.5 \times 10^{-7}$mm, equivalent to $\overline{M}_D \gtrsim 0.78\ \text{TeV}(T_{\text{SN1987A}}/30\ \text{MeV})^{1.3}$, while for four or more dimensions the bounds can be extracted from [26], and are less than a TeV.

A2 In large extra dimension scenarios, there are severe limits on the maximum temperature (the 'normalcy temperature' T_*) above which the evolution of the universe must be non-standard [2]. This temperature is found by equating the rates for cooling by the usual process of adiabatic Hubble expansion, and by the new process of evaporation of KK gravitons into the bulk which becomes dominant at high temperatures. For $\overline{M}_D = 1$ TeV, [2] estimate that $T_* \lesssim 10$ MeV for $\delta = 2$, up to $T_* \lesssim 10$ GeV when $\delta = 6$. Since these temperatures are greater than that at big bang nucleosynthesis, there is no unavoidable constraint on $\overline{M}_D$.

However, as the normalcy temperature is always below the electroweak phase transition temperature, many aspects of early universe cosmology, including baryogenesis and post-inflation reheating, must be carefully rethought.

B1 Distortion of the diffuse cosmic gamma-ray background due to decay on our brane of bulk gravitons to Standard Model states, and in particular two photons. As discussed above supernova cores will also emit large fluxes of KK gravitons, producing a cosmic background of these particles with energies and masses up to about 100 MeV. Radiative decays then give rise to a diffuse cosmic γ-ray background with $E_\gamma \sim 100$ MeV which is well in excess of the observations if more than 0.5–1% of the SN energy is emitted into the new channel [2]. This argument complements and tightens the cooling limit from the observed duration of the SN 1987A neutrino burst. For two extra dimensions a conservative bound [27] on their radius is $R \lesssim 0.9 \times 10^{-4}$ mm, while for three extra dimensions it is $R \lesssim 1.9 \times 10^{-7}$ mm. This corresponds to $\overline{M}_D \gtrsim 29$ TeV and $\overline{M}_D \gtrsim 1.9$ TeV, respectively.

B2 Anomalous heating of neutron stars by the decays of gravitationally trapped Kaluza-Klein graviton modes. In supernovae core collapse formation of neutron stars, massive KK gravitons would be produced with average velocities $\simeq 0.5c$, leading to many of them being gravitationally retained by the supernova core. Thus, every neutron star would have a halo of KK gravitons which decay into e^+e^-, $\gamma\gamma$, and $\nu\overline{\nu}$, on time scales $\simeq 10^9$ years. The EGRET γ-flux limits for nearby neutron stars then lead to the stringent constraint $\overline{M}_D \gtrsim 90$ TeV, for $\delta = 2$, and $\overline{M}_D \gtrsim 5$ TeV for $\delta = 3$ [28]. Even more stringent, the requirement that neutron stars are not excessively heated by KK decays implies $\overline{M}_D \gtrsim 280$ TeV for $\delta = 2$, and $\overline{M}_D \gtrsim 10$ TeV for $\delta = 3$ [28]. This translates into $R \lesssim 9.6 \times 10^{-7}$ mm and $R \lesssim 1.2 \times 10^{-8}$ mm, respectively.

B3 Over production of long-lived massive bulk gravitons leading to so-called overclosure of the universe [2,29]. This leads to a bound $\overline{M}_D > 2.2/\sqrt{h}$ TeV, or $R < 1.5h \times 10^{-5}m$ for 2 extra flat dimensions, where h is the current Hubble parameter in units of 100km/sMpc. Note that the diffuse gamma-ray background limit is more stringent than the overclosure limit, so that the massive KK modes of the bulk graviton can not be the dark matter.

In principle, there are ways to evade the bounds that depend on decay back to our brane [7], even in the flat extra dimension case. For example, there may exist extra brane(s) in the bulk on which gravitons can decay, enhancing $\Gamma_{\rm inv}$. However, extreme parameter values are required.

If there are other fields in the bulk, such as right-handed neutrino states [30,31], then there are other potential astrophysical and cosmological constraints in addition to the ones listed above that must be considered [30,32]. This is particularly true of the scalar field that describes changes in the overall size of the extra dimensions (the 'radion') in the ADD case, as this field is, in all cases so far investigated, light with mass as small as 1 mm^{-1}. Moreover, this field has fixed couplings to the energy momentum tensor. Considerations of early universe cosmology typically lead to a severe radion 'moduli problem,' where coherent excitations of this degree of freedom overclose the universe [33]. Finally the extra spatial dimensions must be frozen in size from at least the big-bang nucleosynthesis epoch onwards, so any late motion of the radion is severely constrained, even if it slow enough to satisfy the overclosure constraint.

Constraints from Precision Electroweak Data

A precise description of the contributions to precision electroweak observables from bulk KK states requires a complete understanding of the underlying theory. Indirect contributions arise from the virtual exchange of KK states, and a summation over the entire KK tower must be performed. This summation diverges for $\delta > 1$ due to the non-renormalizability of the full $4+\delta$-dimensional field theory. In a fully consistent theory, such as string theory, the summation would be regularized and finite. Given our present lack of knowledge of the underlying theory, most authors choose to either terminate the summation with an explicit cut-off set to $\overline{M}_D$, or by invoking flexible branes to exponentially damp the sum [11]. These procedures yield naive estimates of the size of indirect KK contributions, and the resulting constraints are hence merely indicative [34].

Constraints in the special case of TeV^{-1} extra dimensions with $\delta = 1$ and rigid branes have been determined in Refs. 35,36. The contributions to the precision observables from tree-level KK gauge interactions, and from the mixing of the KK gauge states with the SM gauge bosons, have been computed, and a global fit to the data yields [35] a restriction on the compactification radius of $R^{-1} \gtrsim 4$ TeV. In addition, the contribution of the KK gauge states in the fit allow for the Higgs boson to be as heavy as ~ 320 GeV, which is larger than that allowed by the SM electroweak fit. These bounds on R^{-1} can be somewhat reduced in the case of flexible branes [12].

In the RS1 model with SM gauge fields in the bulk, the potential contributions to precision electroweak data depend on the placement of the fermions, on the brane or in the bulk. If the fermions are constrained to the TeV-brane, the couplings of the bulk KK gauge states to the SM fermions are large, being given by $\sqrt{2\pi kR}g \simeq 8.4g$. A global fit to the electroweak data, including bulk KK gauge tree-level and mixing contributions, yields [37] the bound $m_1 > 25$ TeV, which sets $\Lambda_\pi \gtrsim 100$ TeV. This constraint can be relaxed if the fermion fields are also placed in the bulk [19,38].

The contributions of bulk gauge KK states to the anomalous magnetic moment of the muon have been found to be small, whereas bulk fermion KK states can yield potentially sizable shifts in $g-2|_\mu$ in the RS scenario [39].

Graviton contributions to precision observables are notoriously problematic due to the non-renormalizability of the theory. Again, naive estimates on the size of such effects can be obtained in an effective field theory employing a cut-off to regulate the theory. In this approach, the cut-off dependent KK graviton contributions to the electroweak data set are found to be small for $\overline{M}_D \gtrsim 1$ TeV in ADD [40], and disfavor small values of $k/\overline{M}_{\rm Pl}$ in RS1 [19].

Collider Signals

Numerous collider searches have been performed which constrain each of the three scenarios. We limit our discussion to the case of low-energy effective Lagrangians, and note that new collider signatures may also be present in the context of string braneworld models, such as the resonant production of higher spin states [41,42].

Large Extra Dimensions: For the case of large flat extra dimensions, the reactions of individual bulk KK graviton states are not detectable, since they interact with the SM fields on the brane with $\overline{M}_{\rm Pl}^{-1}$ strength. However, the mass splittings between the bulk graviton KK states are given by $\Delta m \simeq 5 \times 10^{-4}$ eV, 20 KeV, and 7 MeV for $\delta = 2, 4$ and 6, respectively for $\overline{M}_D = 1$ TeV, and hence, their number density is large at collider energies. This results in observable signatures at the TeV scale. There are two classes of collider processes: (A) the direct production of bulk graviton KK states and (B) graviton KK production and subsequent decay to wall fields on our brane, *i.e.*, bulk graviton KK virtual exchange. Processes of type (A) are more model independent than those of type (B), but both classes are subject to the restrictions of the effective field theory description below $\overline{M}_D$.

The results of experimental searches for processes of both types are tabulated in the Listings.

A Direct Production of KK Gravitons

This class of collider tests is described by the emission of bulk graviton KK states in scattering processes [14,43] such as $e^+e^- \to \gamma/Z + G_n$, $p\overset{(}{\bar{p}}\overset{)}{} \to g + G_n$, and $Z \to f\bar{f} + G_n$. The graviton appears as missing energy in the detector, behaving as if it were a massive, non-interacting, stable particle. The cross section is computed for a single massive graviton KK state and then summed over the KK tower. This sum can be replaced by an integral weighted by the density of KK states which is cut off by the specific process kinematics. The produced G_n state appears to have a continuous mass distribution corresponding to the probability of emitting KK gravitons with different extra dimensional momenta.

In e^+e^- collisions, the resulting γ/Z angular and energy distributions can be differentiated from those of the SM background reaction $e^+e^- \to \nu\bar{\nu}\gamma$. In addition, if bulk graviton KK emission is observed, then both parameters

$\overline{M}_D$ and δ may be determined by measuring the production rate at different values of $\sqrt{s}$. The cross sections are lowered somewhat in the case of flexible branes [13].

Bulk graviton KK production at hadron colliders results in a mono-jet signal. The effective low-energy theory breaks down for some regions of parameter space, as the parton-level center of mass energy can exceed the value of $\overline{M}_D$. Experiment is then sensitive to the new physics present above this scale. Care must be exercised in interpreting experimental results/simulations, as the effective theory may not be valid over the whole search region.

B Virtual Exchange of KK Gravitons

This class of collider signals consists of bulk graviton KK exchange in all $2 \to 2$ scattering processes [14,44,45]. This results in deviations in cross sections and asymmetries in SM processes, as well as giving rise to new reactions which are not present at tree-level in the SM, such as $gg \to \ell^+\ell^-$. The signature is similar to that expected in the "Quark and Lepton Compositeness" Listings in this *Review*. The exchange process is governed by the effective Lagrangian

$$\mathcal{L} = i\frac{4\lambda}{M_H^4} T_{\mu\nu}T^{\mu\nu} \,. \tag{10}$$

The amplitude is proportional to the sum over the propagators for the bulk graviton KK tower states which, as above, may be converted to an integral over the density of states. However, in this case, the integral is divergent for $\delta > 1$, and thus introduces a sensitivity to the unknown ultraviolet physics. Several approaches to regulate this integral have been employed: (i) a naive cut-off scheme [14,44], (ii) a flexible brane [11], or (iii) the inclusion of full weakly coupled TeV-scale string theory [42]. The most model independent approach is that of the naive cut-off, with the cut-off being set to $M_H \neq M_D$, to account for the uncertainties from the unknown ultraviolet physics. In addition, the parameter $\lambda = \pm 1$ is usually incorporated. Without a full specification of the UV theory, M_H must be treated as a new independent parameter: it cannot be reliably related to

$\overline{M}_D$ and δ. Assuming that the integral is dominated by the lowest dimensional local operator results in a dimension-8 contact-type interaction limit. In the alternate notation of Ref. 14, the coefficient of this effective dimension-8 interaction is $(2/\pi)^{1/4}\Lambda_T = M_H|_{\lambda=+1}$. The resulting angular distributions for fermion pair production are quartic in $\cos\theta$, and thus provide a unique signal for spin-2 exchange.

The simultaneous observation of both classes of processes would signal the existence of large flat extra dimensions, as opposed to other new physics scenarios, as well as determine the parameters of the effective theory.

Once the center of mass energy reaches the scale $\overline{M}_D$, the extra-dimensional gravitational theory described by Eq. (2) becomes strongly coupled, and various exotic production processes might occur. One such possibility is black hole production. Black holes of Schwarzschild radius $R_s \lesssim R$ are substantially altered in the large extra dimension scenario [46], and may have much smaller masses, of order $\overline{M}_D$, than in traditional 4d theories of gravity. Discussions of black hole production at colliders are given in Ref. 47, and constraints on such scenarios from black-holes produced by cosmic rays in the atmosphere are estimated in Ref. 48. Finally, for small-angle elastic scattering, transplanckian collisions, $\sqrt{s} > \overline{M}_D$, can be reliably calculated from the low-energy effective theory and provide additional collider signatures [49].

TeV^{-1}-sized Extra Dimensions: In this scenario, the collider signatures consist of the direct production of the bulk gauge boson KK states at hadron colliders, or the indirect gauge boson KK exchange below production threshold in e^+e^- collisions [50]. This is in direct analogy to the standard signature of new gauge bosons from extended symmetry groups, and is discussed in detail in the extra gauge boson section. The distinction here is that the bulk gauge boson KK coupling strength to the Standard Model fermions is fixed to be $\sqrt{2}g$ for one extra dimension, where g is the SM gauge coupling strength, and the gluon, W, Z, and photon KK states are degenerate, modulo mixing effects. The bounds [35] on the first gauge KK excitation mass are $m_1 > 1.1$ TeV from Drell-Yan production at the Tevatron, and 3.1 TeV from $e^+e^- \to f\bar{f}$ at LEP II. These limits are eclipsed by the constraints from precision electroweak data. If the SM fermions are localized at specific points in the TeV^{-1} dimension, then the exchange of bulk gauge boson KK states in $2 \to 2$ scattering can be used to measure the fermion's wavefunction. The collider signatures of the universal extra dimensions scenario are varied and are discussed in Ref. [17].

Warped Extra Dimensions: In the scenario with warped extra dimensions, the first bulk graviton KK excitation state is of order a TeV, and has TeV^{-1} coupling strength. It can thus be produced as a spin-2 s-channel resonance at colliders. The constraints [18] on the simplest version of the RS model from Run I Tevatron data in the Drell-Yan and dijet resonance channels yield $m_1 \gtrsim 1100\,,600\,,200$ GeV for $k/\overline{M}_{\rm Pl} = 1.0\,,0.1\,,0.01$, respectively. Measurement of the first graviton KK excitation mass and width would determine the two parameters in this model. Higher energy accelerators may be able to directly observe several states of the bulk graviton KK spectrum; measurement of the mass splittings of the KK states would point to the presence of a warped geometry.

If the KK gravitons are too massive to be produced directly, their indirect exchange in fermion pair production results in a contact-like interaction. Unlike ADD, the uncertainties associated with the introduction of a cut-off are avoided in this case, since there is only one additional dimension, and the sum over the KK states thus converges. The sensitivities [18] from LEPII data are $\Lambda_\pi = 4.0, 1.5, 0.4$ TeV, and those from Run I Tevatron searches are $\Lambda_\pi = 3.5, 1.0, 0.35$ TeV for $k/\overline{M}_{\rm Pl} = 0.01, 0.1, 1.0$, respectively.

The production rates for graviton KK states are drastically changed in the scenario where fermions are present in the bulk, and these limits are substantially weaker for some values of the fermion bulk mass parameter [19].

The radion, or graviscalar, is predicted to be the lightest new state in this scenario. Its coupling strength to SM fields is v/Λ_π. The signatures for its production and decay are similar to that of the SM Higgs and are examined in Ref. 51. It is allowed to mix with the SM Higgs, which can result in significant shifts in the properties of the Higgs [52].

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